

# A One-Loop Test of String Duality

Cumrun Vafa

*Lyman Laboratory of Physics, Harvard University*

*Cambridge, MA 02138 USA*

and

Edward Witten

*School of Natural Sciences, Institute for Advanced Study*

*Olden Lane, Princeton, NJ 08540, USA*

We test type IIA-heterotic string duality in six dimensions by showing that the sigma model anomaly of the heterotic string is generated by a combination of a tree level and a string one-loop correction on the type IIA side.

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## 1. Introduction

Six dimensional string theories with non-chiral  $N = 4$  supersymmetry can be constructed most easily either by compactifying the heterotic string on a four-torus or by compactifying a Type IIA superstring on a K3 manifold. According to results of Narain [1] for the heterotic case and of Seiberg [2] and Aspinwall and Morrison [3] for the Type IIA case, these theories have the same moduli spaces of vacua. (Globally, this result depends on mirror symmetry, as was shown in [3], giving an elegant illustration of how conformal field theory behaves in a way that makes space-time stringy dualities possible.) This hints that the two theories might be equivalent, as has been conjectured [4,5].

The hint might appear unconvincing since the equivalence in question is largely determined by the low energy supergravity. However, the equivalence between these theories, which has been called string-string duality, has been supported by a variety of new arguments [6-8]. Also, one of the strangest requirements of the six-dimensional string-string duality, which is that the Type IIA theory must develop massless charged black holes precisely when a cycle in the K3 collapses (see section 4.6 in [6]) has become much more plausible because of dramatic results in four dimensions [9] that depend on an analogous phenomenon. These latter results have been further supported by certain one-loop string calculations [10].

The purpose of the present paper is to subject six-dimensional string-string duality to one small further test. Compactification of the heterotic string on a four-torus gives a six-dimensional theory in which the allowed topology (of the manifold and gauge bundle) is subject to certain restrictions. We would like to verify that the same restrictions also hold in the corresponding Type IIA theory.

One restriction is that the six-manifold must be a spin manifold, since the heterotic string has fermions. This restriction obviously holds, for the same reason, also for the Type IIA theory.

The other known restriction comes from an equation that plays an important role in anomaly cancellation. If  $H$  is the field strength of the two-form,  $F$  the Yang-Mills field strength,  $R$  the Riemannian curvature two-form, and  $\text{tr}$  the trace in the fundamental

representation (of an  $SO(32)$  gauge group or of the Lorentz group  $SO(9, 1)$ )<sup>1</sup> then in the 10 dimensional heterotic string we have

$$dH = -\text{tr}F \wedge F + \text{tr}R \wedge R. \quad (1.1)$$

Upon toroidal compactification of heterotic strings to lower dimensions we get extra gauge symmetries from the left-movers and right-movers; the rank goes up two with each compactified direction. The extra gauge bosons appear in the six-dimensional version of (1.1).

From (1.1) it follows that the cohomology class represented by  $\text{tr}F \wedge F - \text{tr}R \wedge R$  is zero. This cohomology class is, roughly speaking,  $p_1(V) - p_1(T)$ , where  $V$  and  $T$  are the gauge and tangent bundles and  $p_1$  denotes the first Pontryagin class. Actually, a more precise analysis [11] including world-sheet global anomalies shows that this condition can be imposed at the level of *integral* cohomology, not just de Rham cohomology. Also, for a real orientable vector bundle whose structure group can be lifted to the spin group (this is so for  $T$  because fermions exist, and for  $V$  because the heterotic string contains gauge spinors) the first Pontryagin class is divisible by 2 in a natural way. The integral characteristic class obtained by dividing it by 2 seems to have no standard name; we will simply call it  $\frac{1}{2}p_1$ . At any rate, the topological condition in the heterotic string is really

$$\frac{p_1(V)}{2} - \frac{p_1(T)}{2} = 0 \quad (1.2)$$

with the classes understood as integral classes.

In the present paper, we will seek a condition similar to (1.2) for the Type IIA superstring theory compactified on K3. Our analysis will not be precise enough to see the torsion (we comment on this below), but we will see the 2 in the denominator. It is fairly obvious how we must proceed. Since the relation between the field strength  $H$  in the heterotic string and the corresponding field strength  $H'$  for Type IIA is  $H = *e^{-2\phi'}H'$  with  $\phi'$  being the dilaton (the potential importance of such a relation was foreseen by Duff and Minasian [12]), we must replace (1.1) by a relation of the form

$$d^*(e^{-2\phi'}H') = -\text{tr}F \wedge F + \text{tr}R \wedge R. \quad (1.3)$$

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<sup>1</sup> Our gauge bosons are real, antisymmetric matrices, as is natural for  $SO(N)$ , so the trace is *negative* definite.

This will have to be the equation of motion of the two-form field  $B'$ , so the six-dimensional effective Lagrangian must contain a term with the structure

$$\int B' \wedge (\text{tr} F \wedge F - \text{tr} R \wedge R). \quad (1.4)$$

Concretely, if this interaction is present then the desired restriction on the topology of the manifold and gauge bundle will hold because the equations of motion (1.3) have no solution otherwise.

The rest of this paper is devoted to finding the interactions just claimed. It is fairly obvious that the  $\text{tr} F \wedge F$  term must be present at tree level as it is required by low energy supersymmetry. (This term is present since it is dual to the  $\text{tr} F \wedge F$  term in (1.1), which is likewise required by low energy supergravity and so present at tree level.) We explain the details in section 2. The  $\text{tr} R \wedge R$  term is perhaps more mysterious; it arises from a one-loop computation, as we explain in section 3, which gives a term  $\int B' \wedge Y^8$  where  $Y^8$  is a characteristic class involving the Riemann tensor. Compactifying upon K3 leads to a term of the form  $\int B' \wedge \text{tr} R \wedge R$  as is required by the string duality. Note that the presence of this correction at one loop implies an inconsistency in type IIA string compactifications to two dimensions on an eight-manifold with  $\int Y^8 \neq 0$ ; at the one loop level there is no extremum of the effective action.<sup>2</sup> For example if the eight-manifold is  $\text{K3} \times \text{K3}$  the type IIA compactification is destabilized at string one-loop. The corresponding statement for the heterotic side is that if we compactify heterotic string on  $T^4 \times \text{K3}$  without turning on any gauge fields, the heterotic string is inconsistent due to sigma model anomalies. The way to remedy this problem on both sides is to turn on an appropriate gauge field.

One might wonder how one could understand the torsion part of (1.2) for Type II superstrings. Since duality tends to exchange world-sheet and space-time effects, and the torsion shows up in world-sheet global anomalies in the case of the heterotic string, one

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<sup>2</sup> Type IIB compactification on the same manifolds would be inconsistent if we compactify further on a circle because then there would be no distinction between Type IIA and IIB. Also, a similar one loop term which exists for the heterotic string and is responsible for the Green-Schwarz anomaly cancellation mechanism would destabilize compactifications down to two dimensions for which this class is not zero.

might look for space-time global anomalies in Type II that would explain the necessity for the torsion part of (1.2) to vanish. Sometimes global anomalies are more obvious in a soliton or instanton sector than in the vacuum sector. One might in fact look to the solitonic heterotic string of the Type II theory [7,8] as an object whose quantization (like that of the elementary heterotic string) may require vanishing of the torsion part of (1.2).

We must confess to a sin of omission: we have not been precise with orientation conventions and so have not checked the relative sign of the  $\text{tr}F\wedge F$  and  $\text{tr}R\wedge R$  interactions.

We understand that some of the issues in this paper have also been considered in unpublished work by J. Harvey.

## 2. The $\text{tr}F\wedge F$ Interaction

In what follows, we will deduce from ten-dimensional Type IIA supergravity the  $B'\wedge \text{tr}F\wedge F$  interaction. Since the existence of this interaction is certainly already known, the only slightly non-trivial point is to determine the correct normalization; for this we will need a recent result by Harvey and Strominger [8]. Also, in the rest of this paper, we consider only the Type IIA superstring theory, and relabel  $B'$  simply as  $B$ .

In the conventions of [13] (which are used in [8]), the world-sheet coupling involving the  $B$  field is

$$L_B = \frac{1}{4\pi\alpha'} \int_{\Sigma} \epsilon^{ab} B_{IJ} \partial_a X^I \partial_b X^J. \quad (2.1)$$

It follows, in particular, that the period of the  $B$  field is  $4\pi^2\alpha'$ . This means that  $L_B$  is invariant, modulo  $2\pi$  times an integer, under the addition to  $B$  of a closed form  $\beta$  with the property that for every closed surface  $C$  in the target space,

$$\int_C \beta \quad (2.2)$$

is an integer multiple of  $4\pi^2\alpha'$ .<sup>3</sup> We will call a shift  $B \rightarrow B + \beta$  with such  $\beta$  a global world-sheet gauge transformation.

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<sup>3</sup> To be very precise about the meaning of (2.2), if  $C$  is the quotient of the  $x^1 - x^2$  plane by  $x^1 \rightarrow x^1 + 1$ ,  $x^2 \rightarrow x^2 + 1$ , and  $\beta$  has non-zero values  $\beta_{12} = -\beta_{21} = 1$ , then  $\int_C \beta = 1$ .

Ten-dimensional Type IIA supergravity contains a three-form  $C$  with field strength  $G$  – in components  $G_{IJKL} = \partial_I C_{JKL} \pm \dots$  (We will reserve the name  $F$  for the gauge field strengths that will soon appear in six dimensions.) This field couples to  $B$  with a coupling of the general form  $B \wedge G \wedge G$ . Harvey and Strominger normalize  $C$  so that this coupling (in a space-time of Lorentz signature) is

$$-\frac{1}{4\pi\alpha'^3} \int B \wedge G \wedge G. \quad (2.3)$$

With this normalization, they argue that periods of  $G$  are quantized to be integral multiples of  $\alpha'$ . In other words, if  $\Sigma^4$  is any closed four-surface in space-time, then

$$\int_{\Sigma^4} G = n\alpha', \quad \text{with } n \in \mathbf{Z}. \quad (2.4)$$

(For the normalization of such an integral, see the footnote above.) Note that as  $B$  has periods  $4\pi^2\alpha'$ , the existence of the interaction (2.3) implies that  $G \wedge G$  must be  $2\alpha'^2$  times an integral class; (2.4) implies the slightly weaker result that  $G \wedge G$  is  $\alpha'^2$  times an integral class. In compactification to six dimensions on a spin manifold, we will gain an extra factor of two from the fact that the intersection form on the two-dimensional cohomology will be even. The meaning of the factor of 2 in the uncompactified ten-dimensional theory is not clear.

Now, consider the compactification of the ten-dimensional theory to six dimensions on a K3 manifold  $X$ . Let  $U_I$ ,  $I = 1 \dots 24$ , be a basis of harmonic two-forms on  $X$  with integral periods. Then

$$\int_X U_I \wedge U_J = d_{IJ}, \quad (2.5)$$

where  $d_{IJ}$ , the intersection pairing of K3, is even and unimodular with signature  $(4, 20)$  (four positive and twenty negative eigenvalues). Dimensional reduction of the  $C$  field to six dimensions is made by writing

$$C = \frac{\alpha'}{2\pi} \sum_I A^I \wedge U_I, \quad (2.6)$$

where the  $A^I$  are  $U(1)$  gauge fields in six dimensions. The factor of  $\frac{\alpha'}{2\pi}$  in (2.6) is chosen so that, in view of (2.4), the field strengths  $F^I = dA^I$  obey a conventionally normalized

Dirac condition

$$\int_{\Sigma_2} F^I \in 2\pi\mathbf{Z}. \quad (2.7)$$

This condition means that the  $A^I$  can be interpreted as  $U(1)$  gauge fields, coupled to integral charges.

Via (2.6), (2.3) reduces in six dimensions to

$$L_1 = -\frac{1}{4\pi\alpha'} \int B \wedge \sum_{I,J} \frac{d_{IJ} F^I \wedge F^J}{(2\pi)^2}. \quad (2.8)$$

Now, let us verify that (i)  $L_1$  has the correct periodicity with respect to a global world-sheet gauge transformation; (ii)  $L_1$  is minimal in the sense that  $L_1/n$ , with  $n$  an integer  $> 1$ , would not have the right periodicity. Under  $B \rightarrow B + \beta$ ,  $L_1$  changes by

$$\Delta L_1 = -\pi \int \frac{\beta}{4\pi^2\alpha'} \wedge \sum_{IJ} \frac{d_{IJ} F^I \wedge F^J}{(2\pi)^2}. \quad (2.9)$$

This has the properties claimed because  $\beta/4\pi^2\alpha'$  is an arbitrary closed form with integral periods, the  $F^I/2\pi$  are arbitrary forms with integral periods, and a factor of 2 comes from the fact that  $d$  is even; thus  $\Delta L_1$  is an integral multiple of  $2\pi$  but not necessarily an integral model of  $2\pi n$  for any  $n > 1$ .

To express this in terms of the first Pontryagin class, we can split off a sixteen-dimensional sublattice of  $H^2(X, \mathbf{Z})$  on which the intersection form is equivalent to  $-1$  times the usual unimodular, integral form on the weight lattice of  $Spin(32)/\mathbf{Z}_2$ . (One could similarly use  $E_8 \times E_8$ , of course.) If then we interpret the  $F^I$  (restricted to the sixteen-dimensional subspace) as the ‘‘Cartan’’ part of an  $SO(32)$  gauge field, then  $d_{IJ} F^I \wedge F^J$  can be identified with  $\frac{1}{2} \text{tr} F \wedge F$ , the trace now being the trace in the vector representation of  $SO(32)$ . We can then rewrite (2.8) in the form

$$L_1 = -\frac{1}{2\pi\alpha'} \int B \wedge \frac{\text{tr} F \wedge F}{16\pi^2}. \quad (2.10)$$

Here

$$\Theta_V = \frac{\text{tr} F \wedge F}{16\pi^2} \quad (2.11)$$

represents the first Pontryagin class  $p_1(V)$ .

If we write

$$\tilde{B} = \frac{B}{4\pi^2\alpha'} \quad (2.12)$$

– so that  $\tilde{B}$  has integer periods – then (2.11) becomes

$$L_1 = -2\pi \int \tilde{B} \wedge \frac{\Theta_V}{2}. \quad (2.13)$$

This has the right periodicity (it shifts by an integer multiple of  $2\pi$  under global gauge transformations of  $\tilde{B}$ ) because in  $Spin(32)/\mathbf{Z}_2$ , the differential form  $\Theta_V/2$  has integral periods; it represents the class  $p_1(V)/2$  which as mentioned in the introduction is an integral class for bundles that admit spinors.

The discussion so far has been carried out in Lorentz signature. Since the integrand in the Feynman path integral in Lorentz signature is  $e^{iL}$ , while in Euclidean signature it is  $e^{-L_E}$ , the relation between the two (for an interaction such as (2.13) that is independent of the metric and so does not explicitly “see” the signature) is  $L_E = -iL$ , so in Euclidean signature our interaction would be

$$L_1^E = 2\pi i \int \tilde{B} \wedge \frac{\Theta_V}{2}. \quad (2.14)$$

### 3. The $\text{tr}R \wedge R$ Interaction

This section is devoted to finding the six-dimensional  $B \wedge \text{tr}R \wedge R$  interaction whose necessity was explained in the introduction. We first show that in ten dimensional Type IIA, there is a one-loop contribution to the effective action of the form

$$\delta S = \int B Y_8 \quad (3.1)$$

where  $Y_8$  is an eight dimensional characteristic class made of the Riemann tensor contracted with one  $\epsilon$  tensor. The desired six-dimensional interaction then follows upon compactification on K3.

The computation that gives (3.1) is quite similar to familiar computations of anomaly cancellation for the heterotic string [14]. The novelty here is that a somewhat similar term arises for the Type IIA superstring even though this theory is non-chiral. (There is no



such term for the chiral Type IIB theory.) The computational methods of [14] actually directly apply. However for the sake of completeness we will do the computation in our specific case, taking a shortcut in obtaining the final answer.

There are two ways to do the computation: one may, as in [14], compute directly in 10 dimensions the one-loop amplitude involving 4 gravitons and one anti-symmetric tensor field and extract the piece which has the correct index structure; or one may compactify on an eight dimensional manifold  $M$  all the way down to two dimensions and compute the 1 point function of the  $B$  field. The constant of proportionality will be a characteristic class of  $M$  which can be rewritten in terms of the Riemann tensor. The first method is more direct but more difficult. We thus use the second method; applied to the heterotic case this would yield the results already computed in [14].

For Type II strings, one must choose independently even or odd spin structures for left- and right-movers. The  $\epsilon$  tensor in (3.1) will arise in worldsheet computations from the absorption of fermion zero modes, which appear when we have the odd spin-structure. So a term of the form (3.1) can only be generated from computations where the left- or right-movers, but not both, are in an odd spin structure.

In the Type IIB computation the two contributions coming from (even,odd) and (odd,even) cancel out by symmetry, whereas they add in the Type IIA computation. The statements follow from the following. The Type IIA theory is invariant under a parity transformation in space-time combined with a parity transformation of the world-sheet. (To make the Type IIA theory, one uses opposite GSO projections for the left- and right-movers, leading to space-time spinors of opposite chirality. A space-time parity transformation, which exchanges the two types of spinor, must thus be combined with a world-sheet parity transformation.) The interaction (3.1) contains an  $\epsilon$  tensor, which is odd under space-time parity, and a  $B$  field, which is odd under world-sheet parity; altogether (3.1) respects the symmetry of the Type IIA theory. By contrast, the Type IIB theory is invariant under world-sheet parity (unaccompanied by any space-time transformation); this forbids the interaction (3.1), which is odd under  $B \rightarrow -B$ .

### *The Computation*

For the left- or right-moving sector with an odd spin structure there is a supermodulus, which means that we have to use the  $-1$  picture for one of the vertex operators, and in addition insert the supercurrent  $G$  which comes from integration over supermoduli. On the side with even spin structure we will use the picture 0 for the vertex operator. Let us take the left-moving fermions to be in the odd spin structure and the right-moving fermions to be in the even spin structure. The vertex operator for the  $B$ -field which is in picture  $(-1, 0)$  for (left, right) side is

$$V_B = i\delta(\gamma)B_{\mu\nu}\psi^\mu\bar{\partial}X^\nu$$

where we are only interested in the zero momentum contribution so we have set  $k = 0$ . This is the vertex operator normalized so that  $B \in H^2(\mathbf{Z})$ , i.e. has integral periodicity. (This object was called  $\tilde{B}$  in the last section.) Note also that the left moving supersymmetry generator on the worldsheet is

$$G = \psi^\alpha \partial X^\alpha$$

The one loop partition function is obtained by integration over the fundamental domain of

$$\frac{-1}{4} \int_{\mathcal{M}} \langle b(\mu) \bar{b}(\bar{\mu}) [G\delta(\beta)] \bar{c}c V_B \rangle$$

The  $1/4$  factor in front comes from various contributions: a  $1/2$  from  $Z_2$  symmetry of torus, a factor of  $\frac{1}{4}$  from the GSO projection and a factor of 2 because the (odd, even) spin structures would give the same result as (even, odd). Here  $b(\mu)$  is  $b$  folded with the Beltrami differential for the torus, and  $\mathcal{M}$  is the moduli space of a torus with an (even, odd) spin structure pair (this is simply three copies of the fundamental domain without spin structures). The  $b$  and  $c$  insertions simply absorb the ghost zero modes. The superghost zero modes are also absorbed by the superghost delta-function fields (which in the FMS formulation correspond to  $\exp(\pm\phi)$ ). The only way to get a non-zero contribution from the left and right moving  $X$  oscillators is to contract them between left and right using

$$\langle \partial X^\alpha \bar{\partial} X^\nu \rangle = g^{\alpha\nu} \frac{\pi}{\tau_2}$$

where  $\tau$  denotes the moduli on the torus and  $\tau_2$  is its imaginary part. The two left-over fermions are precisely the right number needed to absorb the fermion zero modes on the

worldsheet. Thus all the insertions absorb zero modes of one kind or other, and we are simply left with the partition function. The two-dimensional matter oscillators on the left and right cancel the (super)ghosts on the left and right (except for the zero mode of the bosonic matter field which gives a factor of volume), so we are simply left with the internal partition function for the eight dimensional theory, which is in the (even, odd) spin structure. This is precisely the elliptic genus of the 8 dimensional manifold [15] which we denote by  $A_M(q)$ . Note that it is only a function of  $q = \exp(2\pi i\tau)$  since the right-movers are in the odd spin structure. Collecting all these together we thus have

$$\delta S = \epsilon^{\mu\nu} B_{\mu\nu} \cdot \frac{-i}{4} \int_{\mathcal{M}} \frac{d^2\tau}{2\pi\tau_2} \cdot \frac{\pi}{\tau_2} A_M(q)$$

These are precisely the kind of objects encountered in [14] and the method for integrating over the moduli space is also the same as used there. We write

$$\frac{d^2\tau}{\tau_2^2} = -4i\partial\bar{\partial}\log(\sqrt{\tau_2}\eta\bar{\eta})$$

We then use the fact that the amplitude is total derivative in  $\bar{\partial}_\tau$ , which implies that we only get the boundary contributions. In this limit we are left with the computation

$$\delta S = \frac{-i}{8} \epsilon^{\mu\nu} B_{\mu\nu} \sum \int_{\partial\mathcal{M}} d\tau_1 \left( \frac{1}{\tau_2} + \frac{-4i\partial\eta}{\eta} \right) A_M(q)$$

There are three boundaries of  $\mathcal{M}$ . When we use modular transformation so that they correspond to  $q \rightarrow 0$ , two of them correspond to the NS sector and one corresponds to the  $R$ -sector for right-movers. It suffices to keep the finite piece as  $q \rightarrow 0$  in each of the terms.<sup>4</sup> In particular

$$\frac{1}{\tau_2} + \frac{-4i\partial\eta}{\eta} \rightarrow \frac{\pi}{3}$$

and we replace the  $A_M$  with the massless contribution. In the (NS,R) sector the massless contribution of  $A_M$  computes the index of the Dirac operator coupled to the tangent bundle on  $M$ . Let us call that  $n_{NS,R}$ . In the (R,R) sector the massless contribution to

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<sup>4</sup> In a similar heterotic computation we have to keep a higher order in the  $\eta$  contribution because the elliptic genus has a  $q^{-1}$  term.

$A_M$  computes minus the index of the Dirac operator coupled to the spin bundle on  $M$ . Let us call the index  $n_{R,R}$ . Thus we find

$$\delta S = \frac{i\pi}{24} \epsilon^{\mu\nu} B_{\mu\nu} (2n_{NS,R} - n_{R,R})$$

Note that the relative sign between the  $n_{NS,R}$  and  $n_{R,R}$  is fixed by modular invariance, and they differ by a sign because one is a fermionic state and the other a bosonic one. We thus learn that

$$\int_M Y^8 \sim 2n_{NS,R} - n_{R,R}.$$

$Y^8$  can be expressed in terms of the Riemann tensor if so desired. Actually in the context of 6d string-string duality, we are interested in the piece in  $Y^8$  which is left over after compactification on K3. In particular we are looking for a term in the effective action of the form

$$\delta S = \int B \wedge \tilde{Y}^4$$

where  $\tilde{Y}^4$  is the four form left after integration  $\tilde{Y}^4 = \int_{K^3} Y^8$ . There is only one combination for curvatures which involve one  $\epsilon$  tensor in 4d, and that is just the Pontryagin class given by

$$p_1 = \frac{1}{16\pi^2} \int \text{tr} R \wedge R$$

We can fix the proportionality constant by compactifying further to two dimensions on another  $K^3$ . In this case we find that  $n_{NS,R} = -160$  and  $n_{R,R} = (-16)^2 = 256$  which implies that  $2n_{NS,R} - n_{R,R} = -12 \cdot 48 = 12 \cdot p_1(K_3)$ .

Thus we have learned<sup>5</sup> that for Type IIA string compactified on K3 to six dimension there is a one-loop effective interaction

$$\delta S = 2\pi i \int B \cdot \frac{\Theta_T}{2}$$

where  $B$  has integral periodicity (note  $B = \frac{1}{2} \epsilon^{\mu\nu} B_{\mu\nu}$ ) and  $\Theta_T = \text{tr} R \wedge R / 16\pi^2$  represents the first Pontryagin class of the six-manifold.

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<sup>5</sup> We have further checked the absolute normalization in the above computations using the relation between this computation and the computation of threshold corrections to the theta angle in Type II compactification on  $K3 \times T^2$ .

Note that a similar computation shows that there is no term of the form  $\int B \wedge F \wedge F$  generated at one-loop. This follows simply because all the fundamental string states are neutral under RR gauge fields and the corresponding index contribution would thus vanish. This is just as well, because as discussed above, this term is already present at the tree level for Type IIA strings.

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